

## Collisional Cascades in Planetesimal Disks II. Embedded Planets

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### ABSTRACT

We use a multiannulus planetesimal accretion code to investigate the growth of icy planets in the outer regions of a planetesimal disk. In a quiescent minimum mass solar nebula, icy planets grow to sizes of 1000–3000 km on a timescale  $t_P \approx 15 - 20$  Myr  $(a/30 \text{ AU})^3$ , where  $a$  is the distance from the central star. Planets form faster in more massive nebulae. Newly-formed planets stir up leftover planetesimals along their orbits and produce a collisional cascade where icy planetesimals are slowly ground to dust.

The dusty debris of planet formation has physical characteristics similar to those observed in  $\beta$  Pic, HR 4796A, and other debris disks. The computed dust masses are  $M_d(r \lesssim 1 \text{ mm}) \sim 10^{26} \text{ g } (M_0/M_{MMSN})$  and  $M_d(1 \text{ mm} \lesssim r \lesssim 1 \text{ m}) \sim 10^{27} \text{ g } (M_0/M_{MMSN})$ , where  $r$  is the radius of a particle,  $M_0$  is the initial mass in solids, and  $M_{MMSN}$  is the mass in solids of a minimum mass solar nebula at 30–150 AU. The luminosity of the dusty disk relative to the stellar luminosity is  $L_D/L_0 \sim L_{max}(t/t_0)^{-m}$ , where  $L_{max} \sim 10^{-3}(M_0/M_{MMSN})$ ,  $t_0 \approx 10$  Myr to 1 Gyr, and  $m \approx 1-2$ . Our calculations produce bright rings and dark gaps with sizes  $\Delta a/a \approx 0.1$ . Bright rings occur where 1000 km and larger planets have recently formed. Dark gaps are regions where planets have cleared out dust or shadows where planets have yet to form.

Planets can also grow in a planetesimal disk perturbed by the close passage of a star. Stellar flybys initiate collisional cascades, which produce copious amounts of dust. The dust luminosity following a modest perturbation is 3–4 times larger than the maximum dust luminosity of a quiescent planet-forming disk. In 10 Myr or less, large perturbations remove almost all of the planetesimals from a disk. After a modest flyby, collisional damping reduces planetesimal velocities and allows planets to grow from the remaining

planetesimals. Planet formation timescales are then 2–4 times longer than timescales for undisturbed disks; dust luminosities are 2–4 times smaller.

*Subject headings:* planetary systems – solar system: formation – stars: formation – circumstellar matter

## 1. INTRODUCTION

Observations demonstrate that most stars are born with circumstellar disks of gas and dust. The disks have masses and sizes similar to the protosolar nebula that formed our solar system (Beckwith 1999; Lada 1999). As they age, stars lose their circumstellar disks. In nearby young star clusters, 1 Myr old stars often have massive optically thick disks; 10 Myr old stars rarely have opaque disks (Haisch, Lada, & Lada 2001). However, older stars often have a ‘debris disk’ of dusty material with a size comparable to the radius of the Kuiper Belt of our solar system (Habing et al. 2001; Song et al. 2000; Spangler et al. 2001; Luu & Jewitt 2002). Debris disk masses are at least a few lunar masses and may exceed the mass,  $\sim 0.1 M_{\oplus}$ , of the Kuiper Belt (Backman & Paresce 1993; Artymowicz 1997; Lagrange et al. 2000; Luu & Jewitt 2002; Wyatt, Dent, & Greaves 2003; Greaves & Wyatt 2003).

Recent imaging and photometric data suggest that debris disks often have internal structure. Several disks have asymmetries and warps in their surface brightness distributions (Lagrange et al. 2000; Augereau et al. 2001). Distinct rings of dust surround  $\beta$  Pic (Kalas et al. 2000; Wahhaj et al. 2003),  $\epsilon$  Eri (Greaves et al. 1998), HR 4796A (Jayawardhana et al. 1998; Koerner et al. 1998; Augereau et al. 1999; Schneider et al. 1999; Greaves, et al. 2000), and Vega (Wilner et al. 2002; Koerner, Sargent, & Ostroff 2001). HD 141569 has a dark gap in its disk (Weinberger et al. 1999). Photometric eclipses in KH 15D suggest at least one discrete, opaque object in orbit around the central pre-main sequence star (Herbst et al. 2002). The nearby A-type star Fomalhaut also has a distinct clump in its debris disk (Wyatt & Dent 2002; Holland et al. 2003).

Producing internal structure in a dusty debris disk requires a two (or more) step process. Because radiation removes dust from the disk on timescales shorter than the stellar age, some process must replenish the dust. Collisions between large bodies form copious amounts of dust if the collision velocity is  $\sim 100\text{--}300 \text{ m s}^{-1}$  (e.g. Tanaka et al. 1996; Kenyon et al. 1999; Krivov et al. 2000). Small embedded planets (Kenyon et al. 1999) and close encounters with field stars (Ida et al. 2000; Kalas et al. 2000) can excite large collision velocities and initiate a ‘collisional cascade,’ where 1–10 km planetesimals are slowly ground into fine dust grains. The reservoir of mass needed to maintain a collisional cascade for the 100–500 Myr lifetimes of most known debris disk systems is  $\sim 10\text{--}100 M_{\oplus}$  (Habing et al. 2001; Spangler et al. 2001; Wyatt, Dent, & Greaves 2003). This mass is  $\sim 10\%$  to 100% of the amount of solid material in the ‘minimum mass solar nebula,’ the minimum amount of material needed to form the planets in our solar system scaled to solar abundances (Weidenschilling 1977a; Hayashi 1981).

During the collisional cascade, dynamical processes impose structure on the dust. The gravity of a passing star can produce ring-like density concentrations in a particle disk (Kalas et al. 2000). Radiation pressure ejects small dust grains and forms dusty rings at heliocentric distances of 80–100 AU (Takeuchi & Artymowicz 2001). Small planets stir up leftover planetesimals along their orbits and produce bright rings of dust (Kenyon & Bromley 2002b). Larger planets or shepherd moons can produce dark gaps, partial rings, or warps in the disk (Wilner et al. 2002; Ozerney et al. 2000; Wyatt et al. 1999; Wyatt 2003).

To investigate formation mechanisms for debris disks, we consider dusty disks as plausible remnants of recent planet formation. In the planetesimal theory, planets grow from mergers of smaller objects embedded in the disk (Safronov 1969). Small dust grains in the disk grow to mm sizes and settle into a thin, dusty layer at the midplane of the disk. Collisions between grains in the midplane may form successively larger grains which grow into 1 km ‘planetesimals’ (Weidenschilling 1980; Weidenschilling & Cuzzi 1993; Kornet, Stepinski, & Rózyczka 2001). Planetesimals may form directly through gravitational instabilities (Goldreich & Ward 1973; Youdin & Shu 2003). In either case, slowly-moving planetesimals collide and merge to form larger and larger bodies which eventually become planets (Wetherill & Stewart 1993; Weidenschilling et al. 1997; Kenyon & Luu 1999). The gravity of 1000 km or larger planets stirs up leftover planetesimals to large velocities (Kenyon & Bromley 2001). Collisions between rapidly moving planetesimals produce observable amounts of dust, which often lie in ring-like structures (Kenyon et al. 1999; Kenyon & Bromley 2002b).

In this paper, we continue our study of the bright rings and dark gaps formed during the growth of icy planets in the outer regions of a planetesimal disk. Kenyon & Bromley (2002b) show that icy planets reach sizes of 1000–3000 km on timescales of  $\sim 10$ –20 Myr at 30 AU and 500 Myr to 1 Gyr at 100 AU. The dusty debris of icy planet formation often lies in bright rings with lifetimes of 10–50 Myr. Dark gaps between the bright rings indicate an absence of dust. Here we show that bright, optically thick rings can shadow the disk and produce apparent dark gaps in the disk. The rings and gaps produced in our models are comparable in size and surface brightness to those observed in some debris disks, such as HR 4796A. We also derive predicted lifetimes, luminosities, and masses for collisional cascades induced by embedded planets or a stellar flyby. Although our derived lifetimes are a factor of  $\sim 2$  longer than observed, our results for dust luminosities and masses agree with observations (e.g. Greaves & Wyatt 2003; Habing et al. 2001; Wyatt, Dent, & Greaves 2003; Decin et al. 2003).

We outline the model in §2, describe the calculations in §3, and conclude with a brief discussion in §4.

## 2. THE MODEL

Kenyon & Bromley (2001, 2002a) describe our multiannulus numerical model for planetesimal

growth<sup>1</sup>. Kenyon & Luu (1998, 1999), Kenyon (2002), and Kenyon & Bromley (2001, 2002a) compare results with analytical and numerical calculations. Briefly, we adopt the Safronov (1969) statistical approach to calculate the collisional evolution of an ensemble of planetesimals in orbit around a star of mass  $M$  (see also Spaute et al. 1991; Weidenschilling et al. 1997). The model grid contains  $N$  concentric annuli with widths  $\delta a_i$  centered at heliocentric distances  $a_i$ . Calculations begin with a differential mass distribution  $n(m_{ik})$  of bodies with horizontal and vertical velocities  $h_{ik}(t)$  and  $v_{ik}(t)$  relative to a circular orbit. The horizontal velocity is related to the orbital eccentricity,  $e_{ik}^2(t) = 1.6 (h_{ik}(t)/V_{K,i})^2$ , where  $V_{K,i}$  is the circular orbital velocity in annulus  $i$ . The orbital inclination depends on the vertical velocity,  $i_{ik}^2(t) = \sin^{-1}(2(v_{ik}(t)/V_{K,i})^2)$ .

The mass and velocity distributions evolve in time due to inelastic collisions, drag forces, and long-range gravitational forces. We derive collision rates from kinetic theory and use an energy-scaling algorithm to choose among possible collision outcomes. We define  $S_0$  as the tensile strength of a planetesimal and  $E_g$  as the gravitational binding energy per unit mass. If  $E_c$  is the center of mass collision energy, the ratio  $x_c = E_c/(E_g + S_0)$  sets the collision outcome;  $x_c \ll 1$  yields a merger with negligible debris,  $x_c \sim 1$  yields a merger with some debris, and  $x_c \gg 1$  yields only debris. The ratio of the collision energy  $Q_f$  to the crushing energy  $Q_c$  defines the mass of the debris,  $m_d = Q_f/Q_c$ . For most calculations, we adopt a crushing energy  $Q_c = 5 \times 10^7 \text{ erg g}^{-1}$  and a range of  $S_0$  appropriate for icy objects at large distances from the central star,  $S_0 \sim 1$  to  $10^6 \text{ erg g}^{-1}$  (Greenberg et al. 1984; Kenyon & Luu 1999). Most fragmentation algorithms adopt a minimum velocity for fragmentation,  $V_f$  (Davis et al. 1985; Wetherill & Stewart 1993); we choose  $V_f = 1 \text{ cm s}^{-1}$  (Kenyon & Luu 1999).

We use two algorithms to compute the velocity evolution from long-range gravitational interactions. In the high velocity regime, the collision velocity exceeds the mutual Hill velocity,

$$v_H \approx \Omega_{ij} a_{ij} [(m_{ik} + m_{jl})/3M_\oplus]^{1/3}, \quad (1)$$

where  $\Omega_{ij}$  are  $a_{ij}$  are the average angular velocity and the average heliocentric distance of annulus  $i$  and annulus  $j$ . Statistical solutions to the Fokker-Planck equation then yield accurate estimates for the stirring rates (e.g., Stewart & Ida 2000). In the low velocity regime, the Fokker-Planck equation underestimates the stirring timescales. For some of our calculations, we adopt the Ida & Makino (1993) fits to low velocity stirring rates derived from  $n$ -body simulations. Kenyon & Bromley (2001) describe how we match stirring rates in between the low and high velocity regimes. For other calculations, we adopt the Ohtsuki, Stewart, & Ida (2002) stirring rates derived from fits to the Stewart & Ida (2000) rates and  $n$ -body calculations. In most situations, the Ohtsuki, Stewart, & Ida (2002) rates are 20% to 30% smaller than the Kenyon & Bromley (2001) rates. The Ohtsuki, Stewart, & Ida (2002) rates also agree better with the analytic formulae of Goldreich, Lithwick, & Sari (2002) than the Kenyon & Bromley (2001) rates.

Gas drag circularizes the orbits of all mass batches and also removes material from each batch.

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<sup>1</sup>Th  bault, Augereau, & Beust (2003) describe a single annulus model in the context of the  $\beta$  Pic disk.

We adopt a simple nebular model with gas surface density  $\Sigma_g(a, t) = \Sigma_{g0} a^{-3/2} e^{-t/t_g}$  and scale height  $H_g(a) = H_0 (a/a_0)^{1.125}$  (Kenyon & Hartmann 1987) to compute the gas volume density  $\rho_g$ . To approximate gas removal on a time  $t_g$ , the gas density declines exponentially with time. We set  $t_g = 10$  Myr and adopt the Adachi et al. (1976) formalism to compute inward drift and velocity damping from gas drag (see Weidenschilling 1977b, for another discussion of gas drag). In calculations at 30–150 AU, particle losses from gas drag are small,  $\sim 1\%$  or less of the initial mass. Velocity damping is negligible at late stages when viscous stirring dominates the velocity evolution of small bodies.

The initial conditions for these calculations are appropriate for a disk with an age of  $\sim 1$ –10 Myr. We consider systems of  $N$  annuli in disks with  $a_i = 30$ –150 AU and  $\delta a_i/a_i = 0.01$ –0.025. The disk is composed of small planetesimals with radii of  $\sim 1$ –1000 m (see below). The particles have an initial mass distribution  $n_i(m_{ik})$  in each annulus, with a mass ratio  $\delta = m_{ik+1}/m_{ik}$  between adjacent bins. These objects begin with eccentricity  $e_0$  and inclination  $i_0 = e_0/2$ . Most of our models have  $e_0$  independent of  $a_i$ ; some models have a constant initial horizontal velocity in each annulus,  $e_i \propto a_i^{1/2}$ . We assume a power law variation of the initial surface density of solid material with heliocentric distance,  $\Sigma_i = \Sigma_0 (a_i/a_0)^{-3/2}$ . Models with  $\Sigma_0 \approx 0.1$ –0.2 g cm $^{-2}$  at  $a_0 = 30$  AU have a mass in icy solids comparable to the minimum mass solar nebula (Weidenschilling 1977a; Hayashi 1981). Observed disk masses for 1–10 Myr old stars range from  $\sim 10\%$  to  $\sim 1000\%$  of the minimum mass solar nebula (Osterloh & Beckwith 1995; Natta et al. 1997; Wyatt, Dent, & Greaves 2003).

Our calculations follow the time evolution of the mass and velocity distributions of objects with a range of radii,  $r_{ik} = r_{min}$  to  $r_{ik} = r_{max}$ . The upper limit  $r_{max}$  is always larger than the largest object in each annulus. To save computer time in our main calculation, we do not consider small objects which do not affect significantly the dynamics and growth of larger objects,  $r_{min} = 10$ –100 cm. Erosive collisions produce objects with  $r_{ik} < r_{min}$  which are ‘lost’ to the model grid. Lost objects are more likely to be ground down into smaller objects than to collide with larger objects in the grid.

To estimate the amount of dusty debris produced by planet formation, we perform a second calculation. Each main calculation yields  $\dot{M}_i(t)$ , the amount of mass lost to the grid per annulus per timestep. The total amount of mass lost from the planetesimal grid is  $\dot{M} = \sum_{i=1}^N \dot{M}_i(t)$ . The debris has a known size distribution,  $n'_{ij} = n'_{i0} a_i^{-\beta}$ , where  $\beta$  is a constant. The normalization constant  $n'_{i0}$  depends only on  $\beta$  and  $\dot{M}(t)$ , which we derive at each timestep in the main calculation. To evolve the dust distribution in time, we use a simple collision algorithm. The optical depth  $\tau$  of the dust follows from integrals over the size distribution in each annulus. The optical depth and a radiative transfer solution then yield the luminosity and radial surface brightness of the dust as a function of time. The appendix describes the collision algorithm and the calculations for the optical depth and the dust luminosity.

### 3. CALCULATIONS

#### 3.1. Planet Formation in a Quiescent Disk

##### 3.1.1. Wetherill & Stewart Fragmentation

We begin with calculations in a quiescent disk surrounding a star with a mass of  $3 M_\odot$  and a luminosity of  $50 L_\odot$ . The disk consists of 64 annuli with  $\Delta a_i/a_i = 0.025$  and extends from 30 AU to 150 AU. The initial distribution of icy planetesimals has sizes of 0.5–1000 m with  $\delta = 2$  and equal mass per mass bin. The initial eccentricity is  $e_0 = 10^{-5}$  for all objects in all annuli; the initial inclination is  $i_0 = e_0/2$ . These initial values provide a rough equilibrium between collisional damping and viscous stirring at  $t = 0$ . Viscous stirring dominates at smaller  $e_0$ ; collisional damping dominates at larger  $e_0$ . The particles have mass density  $\rho_d = 1.5 \text{ g cm}^{-3}$ . The initial surface density in solid objects is comparable to the minimum mass solar nebula, with  $\Sigma_0 = 0.18 \text{ g cm}^{-2} (a_0/30 \text{ AU})^{-3/2}$ . The total mass in solids,  $93.8 M_\oplus$ , is similar to the median dust mass observed in disks around young 0.5–3  $M_\odot$  stars (Wyatt, Dent, & Greaves 2003). We adopt the Adachi et al. (1976) gas drag formalism, the Ohtsuki, Stewart, & Ida (2002) stirring rates, and the Wetherill & Stewart (1993) fragmentation algorithm with  $Q_c = 5 \times 10^7 \text{ erg g}^{-1}$  and  $S_0 = 10^6 \text{ erg g}^{-1}$ . These standard choices for  $Q_c$  and  $S_0$  are appropriate for icy planetesimals with bulk properties similar to terrestrial snow (Greenberg et al. 1984).

Icy planet formation in the outer regions of a quiescent disk has three stages (Kenyon & Luu 1999; Kenyon 2002). Small planetesimals with  $r_i \approx 1\text{--}1000 \text{ m}$  and  $e_0 \lesssim 10^{-3}$  grow slowly. At 30–37 AU, it takes  $\sim 0.3 \text{ Myr}$  for the largest objects to grow from  $r_i \sim 1 \text{ km}$  to  $r_i \sim 10 \text{ km}$ . Slow, orderly growth ends when the gravitational cross-sections of the largest objects exceed their geometric cross-sections. Runaway growth begins. The largest objects take  $\sim 2 \text{ Myr}$  to reach sizes of 100 km and another 15 Myr to reach sizes of 1000 km. As the largest objects grow rapidly, dynamical friction and viscous stirring increase the eccentricities of the smallest objects. Collisions between these small objects then begin to produce more and more debris. As the largest objects grow to sizes of 2000–3000 km, a collisional cascade reduces substantially the mass in the smallest objects. During this period of ‘oligarchic growth (Kokubo & Ida 1998),’ the system reaches a rough equilibrium where the largest objects contain most of the remaining mass.

Figure 1 illustrates the time evolution of the cumulative number and eccentricity distributions at 30–37 AU in the inner disk. The initial distributions are  $N_C \propto r_i^{-q_0}$  with  $q_0 = 3$  and  $e_0 = 10^{-5}$  for all objects. During slow growth, the velocity distribution develops two features (Figure 1; right panel). For the largest objects, dynamical friction dominates viscous stirring and produces a power law velocity distribution. At smaller radii, viscous stirring produces a flat velocity distribution with  $e_{\text{flat}}/e_{\text{min}} \sim 0.1$  (Goldreich, Lithwick, & Sari 2002). The transition between these two regimes occurs at the particle radius where most of the mass is concentrated. During runaway growth, this transition moves to larger  $r_i$ ; the shape of the velocity distribution does not change.

The mass distribution consists of two power laws, with a transition at  $r_i \sim 1 \text{ km}$  (Figure 1; left

panel). The largest objects always follow a power law with  $\alpha_l \sim 3$ . At the start of the slow growth phase, collisions produce mergers and little debris. The power law exponent for the small bodies thus decreases from the initial  $\alpha_s \approx 3$  to  $\alpha_s \approx 1$ –2. During runaway growth, debris from collisions adds mass to the smallest mass bins;  $\alpha_s$  slowly increases to the standard collisional exponent,  $\alpha_s \sim 2.5$  (Dohnanyi 1969; Williams & Wetherill 1994). Although collisions between small objects reduce the total mass contained in small bodies, the slope of the mass distribution is roughly constant, with  $\alpha_s \approx 2.5$ .

Figure 2 shows the evolution of the cumulative surface density in the inner disk. During the first 1 Myr of evolution, slow growth concentrates most of the mass in 1–10 km objects. Because most collisions yield mergers with little debris, the total surface density is roughly constant. Runaway growth concentrates more mass in the largest objects. After 10–20 Myr, large objects with radii of 100–1000 km contain roughly 10% of the initial mass in the inner disk. Once oligarchic growth begins, collisions between small objects produce substantial debris instead of mergers. The surface density begins to decline. The largest objects continue to grow and contain an ever-larger fraction of the total mass. After 1–2 Gyr, disruptive collisions between small objects reduce the surface density by nearly an order of magnitude. Large objects with  $r_i \gtrsim 100$  km contain  $\sim 10\%$  of the initial mass and  $\sim 70\%$  of the final mass of solid material at 30–37 AU.

The timescale for planet growth is a strong function of heliocentric distance. Because the collision time is proportional to the surface density and the orbital period, the growth time scales with  $a$  as  $t \propto P/\Sigma \propto a^3$  (Lissauer 1987; Kenyon & Luu 1998). Planetesimals thus grow to large sizes first in the inner disk. In this model, the ratio of timescales for planets to grow to 1000 km at 135 AU and at 33 AU is  $\sim 80$ . This result is close to the predicted ratio of  $\sim 4^3 = 64$ .

Despite differences in timescales, planet formation proceeds to the same endpoint in all annuli. Slow growth, runaway growth, and oligarchic growth produce large objects with  $r_i \sim 100$  km to  $\sim 3000$  km which contain  $\sim 5\%$ – $10\%$  of the initial mass. These large objects stir up the orbits of the leftover planetesimals and initiate a collisional cascade. These disruptive collisions reduce the surface density of small objects by roughly an order of magnitude at all  $a$ .

Erosive collisions also lead to structure in the disk (Figure 3). During the slow growth stages, most collisions yield mergers. The production rate  $\dot{M}$  of objects with  $r_i \lesssim 1$  m is small and declines smoothly with increasing heliocentric distance. As planetesimals merge to form planets, the collisional cascade produces more and more debris. At the inner edge of the disk,  $\dot{M}$  increases by more than 3 orders of magnitude in  $\sim 50$  Myr. The collisional cascade rapidly depletes the inner disk of 1–1000 m bodies;  $\dot{M}$  declines. As large planets begin to form in the outer disk, the peak in  $\dot{M}$  moves to larger heliocentric radii. This peak moves from  $a_i \sim 30$  AU at  $t = 80$  Myr, to  $a_i \sim 50$  AU at 400 Myr, and to  $a_i \sim 100$  AU at  $t = 2$ –3 Gyr.

### 3.1.2. Davis et al. Fragmentation

Calculations with the Davis et al. (1985) fragmentation algorithm allow us to measure the sensitivity of the results to the bulk properties of the planetesimals. In the Davis et al. (1985) approach, the ejecta from a collision receive a fixed fraction  $f_{KE}$  of the center-of-mass collision energy. We follow Kenyon & Luu (1999) and adopt  $f_{KE} = 0.05$  and  $S_0 = 1\text{--}10^6 \text{ erg g}^{-1}$  to simulate a wide range of planetesimal bulk properties (see also Greenberg et al. 1984; Davis et al. 1985).

Planets form faster with the Davis et al. (1985) fragmentation algorithm (Figure 4; see also Kenyon & Luu 1999). During the slow growth phase, collisions that produce some debris tend to reduce  $e$  and  $i$  for the largest planetesimals and increase  $e$  and  $i$  for the smallest planetesimals. Cooling of the large planetesimals increases gravitational focusing factors and speeds the growth of the largest objects during runaway growth. As a result, it takes only  $\sim 15$  Myr for planetesimals to grow to sizes of 1000 km with the Davis et al. (1985) fragmentation algorithm and  $S_0 = 10^6 \text{ erg g}^{-1}$ , compared to  $\sim 20$  Myr with the Wetherill & Stewart (1993) algorithm.

With both fragmentation algorithms, collisional cascades remove nearly all of the mass from a planetesimal disk. Figure 5 compares derived values of  $\dot{M}$  for models with a range of  $S_0$ . In all models, significant debris production begins when large objects with  $r_i \gtrsim 100$  km stir small objects to the disruption velocity,  $V_d \approx 50 \text{ m s}^{-1} (S_0/10^4 \text{ erg g}^{-1})^{1/2}$  (Davis et al. 1985; Kenyon & Bromley 2001). Calculations with small  $S_0$  thus produce more debris at earlier times than calculations with large  $S_0$ . Planetesimals are easier to fragment, but harder to disrupt, in the Davis et al. (1985) fragmentation algorithm than in the Wetherill & Stewart (1993) algorithm. At fixed  $S_0$ , models with Davis et al. (1985) fragmentation produce more debris at early times, and somewhat less debris at later times, than Wetherill & Stewart (1993) models.

Despite these differences, all calculations have several common features. Shortly after the debris production rate reaches a maximum, the decline in  $\dot{M}$  follows a power law with  $\dot{M} \propto t^{-1}$ . At  $t \approx 100$  Myr, all models converge to  $\dot{M} \sim 5 \times 10^{20} \text{ g yr}^{-1}$ . Over a 6 order of magnitude range in  $S_0$ , the dispersion in  $\dot{M}$  at  $t \approx 100$  Myr is smaller than a factor of two. This common  $\dot{M}$  leads to a standard dust mass and disk luminosity at  $t \sim 10\text{--}100$  Myr (see below).

Figure 6 compares final cumulative surface density distributions for the inner eight annuli of several models. For  $S_0 = 10^6 \text{ erg g}^{-1}$ , 1 Gyr of fragmentation reduces the surface density of solid material in the disk by roughly an order of magnitude. For  $S_0 = 1 \text{ erg g}^{-1}$ , the cumulative surface density is roughly a factor of 30 smaller than the initial surface density. Despite a large range in  $S_0$ , the range in the final cumulative surface density is small. In these models, large bodies with  $r_i \gtrsim 100$  km contain  $\sim 1\%$  to  $10\%$  of the initial mass and  $\sim 30\%$  to  $80\%$  of the final mass in solid material.

Stochastic processes are an important feature of the collisional cascade. During runaway growth, random fluctuations in the collision rate can produce a single large body which grows



much more rapidly than bodies in neighboring annuli. By robbing other bodies of material, this runaway body slows the growth of large objects nearby. Stirring by the runaway object leads to more dust on shorter timescales compared to calculations without a single runaway. Single runaways occur in  $\sim 20\%$  to  $25\%$  of test calculations with identical initial conditions. These runaways appear to occur more often in models with smaller  $S_0$ .

The general results of the calculations are insensitive to changes in the initial conditions. For initial eccentricity  $e_0 \leq 10^{-3}$ , all calculations yield growth by mergers instead of rapid disruption (see Kenyon & Bromley 2002a). In models with  $e_0 \sim 10^{-4}$  to  $10^{-3}$ , collisional damping reduces  $e$  until runaway growth begins to produce large objects. Compared to calculations with  $e_0 = 10^{-5}$ , growth times are factors of 2–3 longer (see also Kenyon & Luu 1999). Variations in the initial surface density, the mass density, or the size distribution of planetesimals change the timescale but not the outcome of the evolution. Growth times scale with the initial radius  $r_{i0}$  of the largest object in the grid. For calculations with  $r_{min} = 0.5$ – $1.0$  m, models with  $r_{i0} \sim 10$  m to 1 km have indistinguishable growth times when the slope of the initial power law mass distribution is  $q_0 \lesssim 4$ – $5$ . Because the timescale for viscous stirring is shorter than the growth timescale for  $r_i \gtrsim 10$  km, models with a substantial fraction of the initial mass in 10 km or larger objects take 2–3 times longer to reach runaway growth. Growth rates are more sensitive to the initial mass in solids and the mass density of individual planetesimals,  $t \propto \Sigma^{-1}$  and  $t \propto \rho_g^{-2/3}$  (see Kenyon & Luu 1999).

The results are also insensitive to the details of the gas drag and gravitational stirring algorithms. Because gas drag removes only a few per cent of the initial mass in the disk, changes to the damping or drag algorithm do not change the results. The stirring algorithm affects timescales for the collisional cascade. Calculations with the Ida & Makino (1993) fits for low velocity stirring yield smaller stirring rates than the Ohtsuki et al. (2002) rates. Smaller stirring rates delay the collisional cascade but yield similar debris production. For the range of stirring rates we use in our calculations, the delay in the collisional cascade is  $\sim 25\%$ .

### 3.1.3. Luminosity and Surface Brightness Evolution

Our coagulation calculations demonstrate that planet formation in the outer regions of a planetesimal disk produces copious amounts of dust. The formation of 1000–3000 km bodies leads to a collisional cascade with  $\dot{M} \sim 10^{18}$  g yr $^{-1}$  to  $10^{20}$  g yr $^{-1}$  at 30–100 AU. For dust survival times of  $\sim 1$  Myr at 30–100 AU, these  $\dot{M}$ ’s imply dust masses of  $10^{-3}M_\oplus$  to  $10^{-1}M_\oplus$ . This range is comparable to dust masses derived from observations of debris disks (Backman & Paresce 1993; Lagrange et al. 2000; Zuckerman 2001; Wyatt, Dent, & Greaves 2003). In our simulations, collisional cascades yield large  $\dot{M}$ ’s in narrow ranges of disk radii. The variation in  $\dot{M}$  across the face of the disk (Figure 3) suggests a ring of debris that expands in radius as the formation of 1000–3000 km objects moves to larger disk radii. The apparent dimensions of the ring,  $\delta r_i \sim 20$  AU at  $r_i \sim 50$  AU and  $\delta r_i \sim 30$  AU at  $r_i \sim 100$  AU, are comparable to the dimensions,  $\delta r/r \sim 0.1$ – $0.2$ , of the rings in HR 4796A and Vega (Jayawardhana et al. 1998; Koerner et al. 1998;

Wilner et al. 2002).

To quantify comparisons between our models and observations, we calculate the evolution of particle numbers for ‘dust’ with  $r_i \lesssim 1$  m. The calculation assumes that all small particles have a scale height comparable to the scale height of the 1 m objects in the planetesimal calculation and follows the formation and removal of dust grains by collisions and radiative processes. Gas drag is not included. The evolution of particle numbers yields the radial optical depth through the disk, which we use to derive the time evolution of the radial surface brightness  $I$  and the fraction of stellar radiation  $L_D/L_0$  absorbed and scattered by the disk. This approach complements more detailed calculations of dust in a gaseous disk (e.g., Takeuchi & Artymowicz 2001). Takeuchi & Artymowicz (2001) solve the equation of motion for grains in a gas disk irradiated by a central star but do not include dust formation by collisions. We derive dust production rates from the planetesimal evolution code but do not consider coupling between gas and dust in the disk. The appendix describes the details of our calculation.

Figure 7 shows the evolution of dust mass in several planetesimal disks. We divide the dust into small grains with  $1 \mu\text{m} \lesssim r_i \lesssim 1$  mm and large grains with  $1 \text{ mm} \lesssim r_i \lesssim 1$  m. The small grains have little mass but produce most of the optical depth; the large grains contain most of the mass but have small optical depth. At  $t = 0$ , all disks have no mass in small or large grains. During the slow growth phase, collisions produce modest amounts of debris. The dust mass grows linearly in time. Runaway growth concentrates more and more mass into large objects with  $r_i \gtrsim 100$  km, which stir the leftover planetesimals to the disruption velocity. Once runaway growth begins in the innermost disk annuli, it takes only 3–10 Myr for the dust mass to grow by 4–6 orders of magnitude. The rapid rise in dust mass begins with the formation of objects with  $r_i \gtrsim 100$  km. When the largest objects have  $r_i \sim 1000$  km at  $a_i \sim 30$  AU, the dust mass reaches a plateau. The dust mass is roughly constant until large objects start to form at the outer edge of the disk. Poynting-Robertson drag and radiation pressure then rapidly remove dust from the disk. At late times, the decline in dust mass is  $M_d \propto t^{-n}$ , with  $n \approx 1$ –2.

The mass in small or large grains is remarkably independent of the bulk properties of the planetesimals or the initial conditions in the planetesimal disk. As planet formation propagates through the disk, the dust mass is roughly constant in time. Because planetesimals with small  $S_0$  are easier to fragment than planetesimals with large  $S_0$ , models with  $S_0 = 1 \text{ erg g}^{-1}$  reach this plateau more rapidly ( $\sim 1$  Myr) than models with  $S_0 = 10^6 \text{ erg g}^{-1}$  (30–50 Myr). Despite a six order of magnitude range in  $S_0$ , the range in dust mass is a factor of two for similar starting conditions. Minimum mass solar nebula models with an initial mass of  $\sim 100 M_\oplus$  in solids at 30–150 AU yield a mass in small grains of  $M_s \sim 0.01$ – $0.02 M_\oplus$ ; the mass in large grains is  $M_l \sim 0.1$ – $0.25 M_\oplus$ . The dust mass is equally insensitive to large ranges in  $e_0$  or  $\rho_d$ . The dust mass is roughly proportional to the initial mass of solids in the disk,  $M_s, M_l \propto M_0$ .

The dust masses derived from the planetesimal model are comparable to those observed in debris disk systems. The masses in small grains are close to the minimum mass needed to produce

an observable mid-IR or far-IR excess of radiation for a nearby main sequence star (e.g., Wood et al. 2002). We derive  $M_s \sim 0.01\text{--}0.02 M_\oplus$  in the plateau and  $M_s \gtrsim 10^{-5} M_\oplus$  at late stages of the evolution. Backman & Paresce (1993) quote minimum masses of  $\sim 0.001\text{--}0.01 M_\oplus$  for  $\alpha$  Lyr,  $\alpha$  PsA, and  $\beta$  Pic. For a sample of remnant disks around nearby main sequence stars, Habing et al. (2001) quote minimum dust masses of  $10^{-4}\text{--}10^{-2} M_\oplus$  (see also Greaves & Wyatt 2003; Wyatt, Dent, & Greaves 2003; Decin et al. 2003).

During the collisional cascade, large dust masses result in luminous disks. Figure 8 illustrates the time evolution of the relative disk luminosity  $L_D/L_0$  for models with different  $S_0$ . The left panel shows luminosity evolution for models without radiation pressure on small grains. The right panel shows the luminosity evolution when radiation pressure removes small grains with  $r_i \lesssim 1 \mu\text{m}$  on the local dynamical timescale. Independent of  $S_0$ , models with radiation pressure yield maximum disk luminosities of  $L_D/L_0 \sim 10^{-3}$ . Because radiation pressure is unimportant for large  $S_0$ , the maximum disk luminosity is independent of radiation forces for  $S_0 \gtrsim 10^4 \text{ erg g}^{-1}$ . In models with small  $S_0$ , very small grains ejected by radiation pressure contribute most of the optical depth in the disk. Thus, models without radiation pressure yield lower disk luminosities for  $S_0 \lesssim 10^3 \text{ erg g}^{-1}$ .

Although the magnitude of the disk luminosity is sensitive to the treatment of radiation pressure, the form of the luminosity evolution is independent of  $S_0$  and other parameters in the calculations. All models have a relatively rapid rise in  $L_D/L_0$  followed by a longer decline. While the mass in dust is relatively constant in time, the luminosity follows a shallow power law decline,  $L_D/L_0 \propto t^{-m}$ , with  $m \approx 1$ . As planet formation propagates out through the planetesimal disk, dust forms at larger and larger distances from the central star. Because dust in the outer disk intercepts less radiation from the central star than dust in the inner disk, the dust luminosity declines with time. Once Poynting-Robertson drag removes material from the disk, the luminosity declines more rapidly with a power law index closer to  $m = 2$ .

Collisional cascades also yield a standard maximum luminosity for the outer part of the disk. The luminosity is insensitive to the bulk properties of planetesimals,  $f_{KE}$ ,  $\rho_d$ ,  $Q_c$ ,  $S_0$ , and  $V_f$ , and many of the initial conditions,  $e_0$ ,  $q_0$ , and  $r_{i0}$ . The luminosity is sensitive to the initial mass in planetesimals  $M_0$  at 30–150 AU:

$$\frac{L_D(max)}{L_0} \approx 10^{-3} \left( \frac{M_0}{100 M_\oplus} \right)^{-1}. \quad (2)$$

This luminosity is comparable to the maximum luminosity of known debris disks, such as  $\beta$  Pic and HR 4796A. Because the timescale to reach the maximum luminosity depends on the initial mass in planetesimals, we can write the dust luminosity as a function of time,

$$\frac{L_D}{L_0}(t > t_0) \approx 10^{-3} \left( \frac{t_0}{t} \right)^{-1}, \quad (3)$$

where  $t_0 \approx 30 \text{ Myr } (M_0/100 M_\oplus)^{-1}$ . The coefficient for  $t_0$  ranges from 1–3 Myr for  $S_0 = 1 \text{ erg g}^{-1}$

$\text{g}^{-1}$  to  $\sim 100$  Myr for  $S_0 = 10^6 \text{ erg g}^{-1}$ . Dominik & Decin (2003) derive similar relations from an analytical model for the collisional cascade.

Finally, all planetesimal calculations produce axisymmetric structures within the disk. Figure 9 shows two sets of relative surface brightness distributions for models with different  $S_0$ . For  $S_0 = 10^4 \text{ erg g}^{-1}$  (top panel), the initial surface brightness profile in curve (a) is a power law,  $I/I_0 \propto a_i^{-7/2}$ . During the slow growth phase, the surface area per unit mass of the planetesimals drops and the surface brightness fades. Because planets grow faster in the inner disk, the surface brightness of the inner disk fades more rapidly than the brightness in the outer disk. The surface brightness reaches a minimum at  $t = 4$  Myr;  $I/I_0 \propto a_i^{-7/3}$ . Once runaway growth begins, it takes only 7 Myr for the surface brightness at the inner edge of the disk to reach a maximum roughly two orders of magnitude brighter than the initial planetesimal disk. As planet formation propagates through the disk, a bright ring of dust emission moves outward. This ring highlights the region of maximum dust production and signals the presence of at least one planet with a radius of 1000 km or larger. It takes  $\sim 60$  Myr for this ring to reach  $a_i \sim 50$  AU and another 330 Myr to reach 80 AU. If we define the width  $\delta a/a$  of the ring by the radius where the surface brightness drops to half of the maximum,  $\delta a_i/a_i \approx 0.2$  at  $a_i = 40$  AU and  $\delta a_i/a_i \approx 0.15$  at  $a_i = 80$  AU. At  $t = 400$  Myr, the inner disk is nearly two orders of magnitude fainter than at  $t = 0$ . After 2 Gyr, planet formation reaches the outer edge of the disk and the surface brightness of the entire disk fades. As the disk fades, the surface brightness rises with heliocentric distance, with  $I/I_0 \propto a_i^p$  and  $p \approx 0-2$ .

Calculations with smaller  $S_0$  produce shadowed disks (Figure 9; bottom panel). During the early stages of models with  $S_0 \lesssim 10^2 \text{ erg g}^{-1}$ , the disk surface brightness is a power law and fades slowly with time (curves (a) and (b)). Once runaway growth begins, the inner disk brightens by 2–3 orders of magnitude. The dust in the inner disk is optically thick,  $\tau \approx 3-10$  at  $a_i \approx 30$  AU, and shadows material in the outer disk. Shadowing produces a pronounced minimum in the surface brightness, which propagates outwards as planets form at progressively larger disk radii. Although this shadow resembles the dark gaps cleared of material by a large planet, it is not an absence of dust. The shadow is a region where light from the central star does not penetrate and is therefore of much lower surface brightness than surrounding bright regions with comparable amounts of dust. In models with  $S_0 = 10^2 \text{ erg g}^{-1}$ , it takes only  $\sim 2$  Myr to form a bright inner ring and a dark shadow at  $a_i \approx 30-40$  AU. This structure moves to  $a_i \approx 50-60$  AU at 20 Myr and to  $a_i \approx 80-100$  AU at 100 Myr. The radial extent of the dark gap is  $\delta a/a_i \approx 0.1$ . After 500 Myr to 1 Gyr, planet formation in the outer disk starts to produce dust and the entire disk begins to fade.

Figures 10–11 show a montage of color snapshots of the disk surface brightness. In Figure 10, models with  $S_0 = 10^2 \text{ erg g}^{-1}$  produce a bright ring with a dark shadow outside it. As the ring and shadow propagate out through the disk, they fade relative to the surface brightness of surrounding disk material. In Figure 11, models with  $S_0 = 10^4 \text{ erg g}^{-1}$  produce a bright ring that propagates out through the disk. The ring is brightest where planets with radii of  $\sim 1000-3000$  km form. Inside the ring, planet formation has saturated with the formation of 3000 km objects. Outside the ring, large planets have yet to form.

In the animations of Figures 10–11 included in the electronic version of this paper, planet formation appears as a succession of waves flowing outward in the disk (see also Kenyon & Bromley 2002b). Slow growth from 1 km to  $\sim 100$  km produces little dust and concentrates more and more material into objects with a smaller geometric cross-section per unit mass. Thus, a dark wave moves out through the disk. Dust formed during runaway growth lies in bright rings which appear as a bright wave moving out through the disk. Once planet formation is complete, a dark wave – which heralds the disappearance of dust – propagates out through the disk. During this phase, the largest bodies in the disk may coalesce to form larger planets.

### 3.2. Planet Formation After a Stellar Flyby

Planetesimal calculations demonstrate that embedded planets with  $r_i \gtrsim 1000$  km can produce bright rings and dark gaps in a quiet planetesimal disk. Stellar flybys can also produce rings, gaps, and possibly other structures in a planetesimal disk (Larwood 1997; Mouillet et al. 1997; Kalas et al. 2000; Ida et al. 2000; Kobayashi & Ida 2001; Larwood & Kalas 2001). The lifetimes for rings and gaps produced by a flyby are much shorter,  $\lesssim 1$  Myr, than the lifetimes of rings and gaps produced by planets,  $\sim 10$ – $100$  Myr. Because field stars are less likely to interact with passing stars than stars in young associations, flybys are more relevant to understanding the structures observed in young debris disks than in old debris disks (e.g. Ida et al. 2000; Kalas et al. 2000; Kenyon & Bromley 2002a). Here we consider whether planet formation can regenerate bright rings and gaps following a stellar flyby.

Because our code does not currently follow the trajectories of individual objects, we assume that the close passage of a low mass star at  $a \gtrsim 600$  AU instantaneously raises the eccentricities of planetesimals. This approximation is valid if the encounter between the passing star and the disk is short compared to the collisional damping time. Passing stars not bound to the central star of the disk satisfy this requirement (Larwood 1997; Mouillet et al. 1997). We adopt a functional form for the imposed eccentricity that satisfies constraints derived from  $n$ -body calculations (Larwood 1997; Mouillet et al. 1997; Kobayashi & Ida 2001).

Kenyon & Bromley (2002a) describe the early evolution of a planetesimal disk following a moderately close stellar flyby. Large perturbations with  $e_0 \gtrsim 0.05$  lead to complete disruption of nearly all planetesimals (Kenyon & Bromley 2002a, and references therein). When a flyby produces a modest perturbation with  $e_0 \lesssim 0.03$ – $0.04$  at 30–150 AU, two body collisions produce substantial amounts of dust and damp planetesimal velocities on short timescales. Once planetesimals have  $e \lesssim 0.01$ , collisions produce growth instead of disruption. Continued damping and growth eventually produces planets.

To illustrate planet formation in a planetesimal disk following a stellar flyby, we consider in detail a disk model with  $e_0 = 0.02$  ( $a_i/a_0$ ) $^{1/2}$ ,  $\Sigma_0 = 0.02$  g cm $^{-2}$  at  $a_0 = 30$  AU, and a total mass in solids of  $9.3 M_\oplus$ . This disk mass is at the lower end of the observed range of dust masses for

0.5–3  $M_{\oplus}$  stars (Wyatt, Dent, & Greaves 2003). The disk consists of 64 annuli with  $\Delta a_i/a_i = 0.025$  and extends from 30 AU to 150 AU. The initial distribution of planetesimals has sizes of 10 cm to 10 m with  $\delta = 2$  and equal mass per mass bin. The particles have mass density  $\rho_d = 1.5 \text{ g cm}^{-3}$ . We adopt the Kenyon & Bromley (2001) stirring algorithm and the Wetherill & Stewart (1993) fragmentation algorithm with  $Q_c = 5 \times 10^7 \text{ erg g}^{-1}$  and  $S_0 = 2 \times 10^6 \text{ erg g}^{-1}$ . Kenyon & Bromley (2002a) describe several aspects of the early evolution of this model.

Planet growth following a stellar flyby has four stages. After the flyby, planetesimals damp quickly and begin to form larger objects. This evolution starts at the inner edge of the disk and slowly propagates outward. At all heliocentric distances, the 50–200 cm planetesimals damp first. This damping produce a V-shaped eccentricity distribution in  $\lesssim 1 \text{ Myr}$  at 30–37 AU (Figure 12). After 2–4 Myr, larger planetesimals damp and begin to grow slowly. As objects grow to sizes of 0.1–1 km, dynamical friction dominates viscous stirring and reduces the eccentricities of the largest objects.

When particles reach sizes of 1 km or larger, the growth and stirring rates increase rapidly (Figure 13). Damping of the eccentricities of the largest objects leads to runaway growth. At 30–37 AU, it takes 40 Myr for the largest objects to reach sizes of 10 km and another 20 Myr for large objects to reach sizes of 100 km. These timescales are a factor of  $\sim 2$  larger than growth times for icy planets in a quiet disk. Viscous stirring by the largest bodies increases the eccentricities of the smallest objects. The velocity evolution quickly reaches an equilibrium between the largest bodies with radii of 100 km or larger and 1–10 km bodies containing most of the mass. This ratio is roughly  $e_{\text{large}}/e_{\text{small}} \sim 0.03\text{--}0.1$  (Goldreich, Lithwick, & Sari 2002). As the largest bodies grow to sizes of 1000 km, they stir up the smallest bodies to this equilibrium eccentricity ratio.

All processes take longer farther out in the disk. The timescales for collisions, damping, and stirring grow with heliocentric distance. The collision time is  $t \propto P/\Sigma \propto a^3$ , which implies a damping timescale of  $\sim 0.3 \text{ Myr}$  at 30–37 AU and  $\sim 25 \text{ Myr}$  at 130–150 AU. Because our model has  $e_0 \propto a_i^{1/2}$ , the outer disk damps more slowly and loses more material than the inner disk. This extra mass loss slows the rate of planet formation by another 20% to 30% relative to the inner disk. Thus, planet formation takes  $\sim 200$  times longer at 150 AU than at 30 AU.

Throughout the evolution, the cumulative size distribution has three main components (Figure 14). As collisions damp particle eccentricities, fragmentation dominates growth by mergers. The size distribution for the small objects follows a power law with  $\alpha_s \approx 2.5$  for  $r \lesssim 0.1\text{--}1 \text{ km}$ . Once mergers become important, the size distribution develops a merger component, with  $\alpha_l \approx 3$  for  $r \gtrsim 1\text{--}10 \text{ km}$ . At intermediate sizes,  $0.1 \text{ km} \lesssim r \lesssim 10 \text{ km}$ , the size distribution has a pronounced hump containing most of the mass. As planets grow in the disk, the position of this hump moves to larger radii.

The right panel of Figure 13 illustrates some features of the collisional cascade induced by planet formation. During the early evolution of this model, erosive collisions remove material from the disk. The cumulative surface density declines by  $\sim 30\%$  at 30–37 AU and 40% to 50% at 125—

150 AU. As mergers produce 1–100 km objects, the cumulative surface density is roughly constant in time. Once 100–1000 km objects begin to form, viscous stirring increases the eccentricities of the leftover planetesimals (Figure 12). Continued stirring leads to the collisional cascade, where 0.1–1 km planetesimals are slowly ground into smaller and smaller objects. Because 1–10 km planetesimals are too strong, the hump in the size distribution shifts from  $\sim 1$  km to  $\sim 10$  km. After 2 Gyr, the collisional cascade produces a factor of two decline in the cumulative surface density.

Planet growth and the collisional cascade depend on the initial mass in planetesimals. More massive disks have shorter collision times, with  $t_c \propto M_d^{-1}$  (e.g., Lissauer 1987; Kenyon & Luu 1998, 1999). In a disk with a mass in solids comparable to the minimum mass solar nebula, planets grow a factor of ten faster than in the model in Figures 11–13. Figure 14 shows that the character of the evolution is not sensitive to the initial mass. The cumulative number distribution rapidly develops a smooth power law for  $r \lesssim 0.1$  km, with  $\alpha_s \approx 2.5$ . For  $r \gtrsim 10$  km, mergers lead to a steeper power law with  $\alpha_l \approx 3$  on longer timescales. After  $\sim 30$  Myr, the largest objects have  $r \sim 1000$  km at 30–37 AU. The collisional cascade begins and reduces the surface density by a factor of 3 over the next 300 Myr. After 3 Gyr, erosive collisions reduce the surface density at the inner edge of the disk by another factor of three. The largest objects then have radii of  $\sim 2000$ –3000 km; most of the mass is in objects with  $r \sim 10$ –100 km.

To test the robustness of these results, we calculated models with a variety of initial conditions. We changed  $S_0$ ,  $e_0$ , and the initial variation of  $e$  with  $a$ . Besides the initial mass in planetesimals, the ratio  $e_0/S_0$  is the important parameter in these calculations. Compared to our baseline models with  $e_0/S_0 = 10^{-8}$ , models with  $e_0/S_0 > 10^{-8}$  lose a larger fraction of their initial mass and take longer to form planets. When  $e_0/S_0 \gtrsim 3 \times 10^{-8}$ , erosive collisions remove almost all planetesimals before collisional damping becomes effective. These calculations do not form planets on reasonable timescales,  $\lesssim 1$  Gyr. For calculations with fixed  $e_0/S_0$ , the mass of the largest planet scales with  $S_0$ . In our models, collisional cascades begin sooner when  $S_0$  is smaller. Because the collisional cascade robs material from the largest bodies, smaller  $S_0$  prevents large objects from growing.

Calculations with the Davis et al. (1985) fragmentation algorithm yield similar results. Our tests show that damping is more efficient in calculations where  $f_{KE}$  is large and less efficient when  $f_{KE}$  is small. In general, icy planetesimals with  $f_{KE} \approx 0.03$ –0.10 are harder to disrupt and easier to fragment in the Davis et al. (1985) algorithm. Thus, these models tend to produce less debris and to form planets on shorter timescales than our baseline models. This difference is small,  $\sim 25\%$  to  $35\%$ .

Stellar flybys and planet formation produce copious amounts of dust. Figure 16 compares the evolution of the debris production rate  $\dot{M}$  of flyby models with the  $\dot{M}$  evolution for a quiet disk model. During the flyby, collisions produce substantial debris on a short timescale. The derived  $\dot{M}$  is 6–7 orders of magnitude larger than the initial  $\dot{M}$  for a quiescent disk and 4–5 orders of magnitude larger than the largest  $\dot{M}$  for a planet-forming disk. Collisions damp planetesimal velocities;  $\dot{M}$

declines. As collisions start to favor mergers over debris production, the decline in  $\dot{M}$  accelerates. When 10 km objects begin to form in the inner disk, the global  $\dot{M}$  reaches a minimum. The growth of 100 km objects leads to a linear increase in  $\dot{M}$ ; the growth in  $\dot{M}$  accelerates once 1000 km objects begin to form and stir up the leftover planetesimals. Debris production saturates during oligarchic growth and then declines as planet formation propagates out through the disk.

Compared to quiet disk models, planet formation following a stellar flyby produces less dust at later times (Figure 16). After a stellar flyby perturbs a planetesimal disk, planets grow more slowly; once planet formation begins, the reservoir of leftover planetesimals is smaller. For a modest perturbation with  $e_0 = 0.02$ , dust production following a flyby begins 3–10 times later and peaks at a factor of two smaller  $\dot{M}$  than dust production in a quiet disk. Because A-type stars evolve into red giants on 1 Gyr timescales, stronger perturbations probably prevent planet formation at 30–150 AU around A-type stars.

We conclude this section with a brief discussion of Figure 17, which illustrates the evolution of the dust luminosity following a stellar flyby. The initial perturbation produces a substantial dust luminosity that is 2–4 times larger than the maximum luminosity of a planet-forming disk (compare with Figure 8). This large luminosity is short-lived and declines rapidly as collisions cool the planetesimal swarm. As objects grow to 10 km sizes in the inner disk, the dust luminosity reaches a minimum roughly 3 times more luminous than the luminosity minimum for a quiescent planet-forming disk. Dust production associated with planet formation leads to a rise in luminosity followed by a linear decline as in the quiet disk models. The peak luminosity in flyby models is significantly smaller than in quiet disk models. For similar initial masses, flyby models with  $e_0 = 0.02$  are a factor of 3–5 fainter than quiet disk models. The flyby also causes a substantial delay in the rise in dust luminosity. Flyby models with  $e_0 = 0.02$  reach maximum dust luminosity 5–10 times later in time than quiet disk models.

### 3.3. Limitations of the Models

In previous papers, we have described limitations to multiannulus (Kenyon & Bromley 2001, 2002a) and single annulus (Kenyon & Luu 1998, 1999) coagulation calculations. Here, we summarize the most important of these limitations and consider uncertainties in the dust calculation.

As long as the statistical assumptions underlying the formalism are met, coagulation calculations provide a reasonable representation of real collision evolution (Wetherill 1980; Greenberg et al. 1984; Davis et al. 1985; Barge & Pellat 1991; Spaute et al. 1991; Lissauer & Stewart 1993; Wetherill & Stewart 1993; Stern & Colwell 1997; Weidenschilling et al. 1997; Kenyon & Luu 1998; Inaba et al. 2001). In our calculations, the spacing of mass bins in an annulus and the spacing of annuli in the disk limit the accuracy of the results. Our choice of mass spacing,  $\delta = 2$ , lengthens the evolution time by 10% to 20% (see Kenyon & Luu 1998, and references therein). The radial resolution in our grid of annuli,  $\Delta a_i/a_i = 0.025$  also lengthens the evolution time. Tests



with different grids suggest a lag of 15% to 25% relative to calculations with very fine grids. Thus, our evolution timescales are  $\sim 30\%$  to  $40\%$  longer than the actual evolution times.

If  $N$  is the number of annuli and  $M$  is the number of mass batches, the computation times increase roughly as  $N^2M^2$ . Because a typical calculation requires 2–4 weeks of cpu time, increasing the resolution of a calculation to achieve a more accurate evolution time is prohibitively expensive. Improving the accuracy of our evolution times requires practical increases in computing speed, which will be achieved with the next generation of high-speed parallel computers.

The coagulation algorithm begins to break down when binary interactions between large objects become important. We reach this limit when  $r_i \approx 1000\text{--}2000$  km. At this point, the coagulation algorithm underestimates collision and stirring times, and overestimates the evolution time for dust clearing and the growth of the largest objects. Tests with a hybrid  $n$ -body–coagulation code (Bromley & Kenyon 2003) suggest that our pure coagulation results overestimate the dust clearing time by less than a factor of two. Although the hybrid code forms larger objects than the pure coagulation code, differences in the final mass distribution are small. We plan to report on these calculations in a future paper.

In our implementation, the inherent limitations of the coagulation algorithm have clear observational consequences. We overestimate the timescale to produce large planets by  $\sim 40\%$  and the timescale for dust clearing by a factor of  $\sim 2$ . Because the collisional cascade must remove the same amount of material on a shorter timescale, our estimates for dust mass and luminosity are probably low by a factor of  $\sim 1.5\text{--}2.0$  compared to ideal calculations with perfect resolution in mass and radius. Our bright dust rings are probably  $50\%$  larger in radius than those in an ideal calculation; our dark gaps are probably smaller by a similar factor. If the actual clearing time for dust is shorter, then our radial surface brightness profiles are too shallow during the late stages of the evolution. All of the uncertainties in timescales are small compared to the 1–2 order of magnitude range in evolution timescales set by the range in initial disk mass (e.g. Wyatt, Dent, & Greaves 2003) and the bulk properties of planetesimals. Thus, the observational uncertainties are set more by unknown physics than by limits in the calculations.

Our estimates for the evolution of dust mass, luminosity, and surface brightness rely on several assumptions, (i) large bodies do not accrete small grains, (ii) small grains have the same scale height as 1 m objects, and (iii) the size distribution of dust grains is fixed at  $N_c \propto r_i^{-5/2}$ . Large bodies probably accrete  $\sim 1\text{--}3\%$  of the total mass converted to dust, which has a negligible impact on the evolution of small or large bodies. Stirring by large planets sets the scale height of the smallest objects in the planetesimal grid. The amount of stirring does not depend on the mass of the small objects. Because gas drag is unimportant at late stages in our calculations, the scale height of small objects is fairly independent of their mass (Figures 1, 4, and 8). We estimate a factor of  $\sim 2$  uncertainty in the scale height of the small objects, which yields factor of two uncertainties in the dust luminosity. Dohnanyi (1969) and Williams & Wetherill (1994) show that collisional cascades produce power law size distributions with  $\alpha_s \approx 2.5$  (see also Tanaka et al. 1996). The

uncertainty in this exponent is small,  $\pm 0.05$  or less for a large range in input parameters. In our calculations, significant deviations from this power law can occur at a boundary size,  $r_b$ , where collisions produce complete disruption for  $r_i \lesssim r_b$  and modest fragmentation for  $r_i \gtrsim r_b$ . For the bulk properties assumed in our calculations,  $r_b > 1$  m, the minimum size of planetesimals in our main calculation. Thus we expect  $\alpha_s \approx 2.5$  for small particles to a high degree of accuracy. This result yields a small uncertainty,  $\lesssim 5\%$ , in the derived dust masses and luminosities.

#### 4. DISCUSSION AND SUMMARY

Our calculations demonstrate that icy planet formation is the inevitable outcome of coagulation in the outer regions of a planetesimal disk (see also Lissauer 1987; Kenyon 2002). In a quiet disk, icy planets with  $r_i \approx 1000$ – $3000$  km form on a timescale

$$t_P \approx 15 - 20 \text{ Myr} \left( \frac{\Sigma_0}{\Sigma_{MMSN}} \right)^{-1} \left( \frac{a}{30 \text{ AU}} \right)^3, \quad (4)$$

where  $\Sigma_{MMSN}$  is the surface density of the minimum mass solar nebula. Planets form more slowly in a disk perturbed by a stellar flyby. Large perturbations prevent planet formation. In modest perturbations with  $e_0 \approx 0.02$ – $0.04$ , planet formation timescales are a factor of 2–4 longer than timescales in a quiet disk.

Icy planet formation produces copious amounts of dust. As planets grow to sizes of 100–1000 km, gravitational stirring increases the orbital eccentricities and inclinations of leftover planetesimals. High velocity collisions between leftover planetesimals lead to a collisional cascade, where 1 km objects are slowly ground to dust. The collisional cascade reduces the surface density in the disk by 90%–95%. The reduction in surface density depends weakly on the tensile strength  $S_0$  of 1 km planetesimals. Collisions between ‘weak’ planetesimals with  $S_0 \lesssim 10^3 \text{ erg g}^{-1}$  produce more dust than collisions between ‘strong’ planetesimals with  $S_0 \gtrsim 10^3 \text{ erg g}^{-1}$ .

In all of our calculations, dust is confined to bright rings separated by dark gaps. Bright rings form along the orbits of large planets, which stir leftover planetesimals up to the disruption velocity. Dark gaps are regions with little dust, where planets have not yet formed, or regions shadowed by bright, optically thick dusty rings in the inner disk. The bright rings and dark gaps have sizes, 0.1–0.2  $a$ , comparable to the rings and gaps observed in HR 4796A and other debris disks.

Dusty rings produced by icy planet formation are observable. The maximum luminosity of a dusty disk, equation (2), is comparable to the dust luminosities of known debris disk systems (Backman & Paresce 1993; Kalas 1998; Lagrange et al. 2000; Habing et al. 2001; Spangler et al. 2001; Greaves & Wyatt 2003; Decin et al. 2003). The decline in dust luminosity follows a power law,  $L_d \propto t^{-m}$ , with  $m \approx 1$ – $2$  (Dominik & Decin 2003, see also equation (3)). If the initial mass in planetesimals is comparable to the mass of a minimum mass solar nebula, the timescale for the decline in dust luminosity is 500 Myr to 2 Gyr. More massive disks evolve on shorter timescales. The maximum dust luminosity agrees with observed luminosities of debris disks. Our

derived lifetimes are longer than the observed lifetimes (see also Greaves & Wyatt 2003; Decin et al. 2003). Limitations in our calculations probably lead us to overestimate the decline timescales by a factor of  $\sim 2$ –3. Given the uncertainties in the initial conditions, this ‘correction’ brings our model lifetimes into reasonable agreement with observations.

Aside from the initial disk mass and the mass density of planetesimals, our results are insensitive to most input parameters. The dust mass – and thus the dust luminosity and surface brightness – does not depend on the bulk properties or the initial orbital or mass distributions of the planetesimals. The evolution timescales are longer if planetesimals are more dense, stronger, and have larger initial orbital eccentricities. Porous, weaker planetesimals in more circular orbits grow more rapidly. For fixed initial conditions, the uncertainties in the results are small compared to the 1–2 order of magnitude range in the initial disk mass (Wyatt, Dent, & Greaves 2003).

Together with other studies, our results demonstrate that several physical processes can produce rings, gaps, and other structures in a debris disk. If the gas density in the disk is modest, coupling between the gas and the dust can produce rings and gaps (Takeuchi & Artymowicz 2001). Resonant interactions between dust and distant large planets (Wilner et al. 2002; Kuchner & Holman 2003) or small moons (Wyatt et al. 1999) can produce rings or ring arcs in the disk. Large planets can clear large gaps in the disk (Lin & Papaloizou 1979; Goldreich & Tremaine 1980). Our models produce bright rings from collisional cascades induced by small planets within the ring. Dark gaps arise between small planets in nearby bright rings or by shadowing of the disk by an optically thick bright ring.

In all models, the visibility of dusty structures in a debris disk is related to the collision rate. The collision lifetime of a single particle of radius  $r$  in an annulus of width  $\Delta a$  at distance  $a$  from the central star is  $t_c \approx (dn/dt)^{-1}$ :

$$t_c \approx \frac{a \Delta a}{\Omega n r^2}, \quad (5)$$

where  $\Omega$  is the orbital frequency and  $n$  is the number of particles. The luminosity of the annulus is roughly  $L_D/L_0 \approx \tau H/a \approx (r/a)^2 n$ , which yields

$$t_c \sim 10 \text{ Myr} \left( \frac{\Delta a}{a} \right) \left( \frac{P}{1000 \text{ yr}} \right) \left( \frac{L_D/L_0}{10^{-5}} \right)^{-1}. \quad (6)$$

Because the optical depth and collisional cross-section depend on  $r^2$ , the collision time is independent of particle size. Most debris disks have relative luminosities  $L_D/L_0 > 10^{-5}$ , orbital periods  $P < 1000$  yr, and relative sizes  $\delta a/a \approx 0.1$ –0.2. The collision time of 10 Myr or less is shorter than the typical stellar age,  $t \gtrsim 10$ –100 Myr. Particle collisions thus play an important role in the evolution of all known debris disks. Collisions are most important in the most luminous debris disks such as  $\beta$  Pic and HR 4796A.

As samples of debris disk systems grow, observations might measure the importance of collisions. In bright disks, very small grains ejected by radiation pressure produce a large optical depth. Systematic variations in the radial scale height with disk luminosity or stellar age might

imply some emission from these small grains. Measured disk luminosities inside bright rings might constrain the amount of material dragged inwards by Poynting-Robertson drag and thus the mass in small grains inside the bright ring. As our calculations become more sophisticated, we plan to make detailed predictions for comparisons with observations.

Our results suggest caution when interpreting bright and dark structures in debris disks (see also Takeuchi & Artymowicz 2001). Dark gaps can indicate a large planet (Lin & Papaloizou 1979; Goldreich & Tremaine 1980), with a mass  $m_p \approx 3(\Delta a/a)^3 M_\star$ . For the measured sizes of dark gaps in known debris disks,  $m_p/M_\star \lesssim 10^{-3}$ . In our models, shadowing or several bright rings can produce dark gaps with  $\Delta a/a \approx 0.01$ -0.1. These structures imply smaller planets with  $m_p/M_\star \lesssim 10^{-7}$ . In systems with residual gas, interactions between the gas and small dust grains can produce a single bright ring (Takeuchi & Artymowicz 2001) without any planet.

Observations can plausibly distinguish between these possibilities. Multiple rings, apparently observed in  $\beta$  Pic (Wahhaj et al. 2003), favor planet formation over dust-gas interactions. Measured scale heights or velocity dispersions might distinguish between large or small planets associated with a bright ring or a dark gap. Large scale heights with  $H/a \approx 0.1$  are more consistent with Jovian mass planets; small scale heights with  $H/a \approx 10^{-2}$  imply smaller planets with masses ranging from Pluto to the Earth.

Finally, multiannulus accretion codes are an important step towards understanding the formation and evolution of planetary systems (see also Spaute et al. 1991; Weidenschilling et al. 1997). Our calculations demonstrate that ‘complete’ disk models with 64 or more annuli provide interesting conclusions regarding icy planet formation in a planetesimal disk. As improvements in computers allow larger simulations, we plan to extend these calculations into the terrestrial and giant planet regimes to improve constraints on the planetesimal theory. These calculations also make interesting predictions regarding outcomes of planet formation. Observations with satellites – e.g., *SIRTF* and *JWST* – and ground-based telescopes – e.g., *OWL* and *GSMT* – will provide tests of these predictions and lead to improvements in our understanding of planet formation.

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## A. APPENDIX

Our planetesimal calculations derive the evolution of the numbers and orbital elements for objects with radii of 1 m and larger as a function of time (Kenyon & Bromley 2001, 2002a). Detailed calculations for the evolution of the smaller particles is prohibitively expensive in computer time. To estimate the number of smaller particles, we perform a second, approximate calculation that uses the production rate  $\dot{M}$  of small particles derived in the planetesimal calculation<sup>2</sup>. Throughout the later stages of the planetesimal calculations,  $h_{ik} \approx \text{constant}$  for 1 m to 1 km objects. We thus assume that the scale height of small particles is identical to the scale height of 1 m objects in the planetesimal grid. During a collisional cascade, Dohnanyi (1969) and Williams & Wetherill (1994) show that the cumulative number distribution of small objects is a power law,  $N_C = N_0 r^{-\alpha}$ , with  $\alpha \approx 2.5$ . At late times, the 1 m to 1 km objects in our planetesimal calculations reproduce this result. We therefore assume that the dusty debris produced in the planetesimal calculations has the same power law. The dust production rate in each annulus  $\dot{M}_i$  thus yields the normalization constant  $N_0$  of each annulus for each timestep.

We consider two populations of dust, very small grains and larger grains. On a dynamical time scale set by the local circular velocity,  $v_K = (GM/a)^{1/2}$ , radiation pressure ejects very small grains with radii between  $r_1$  and  $r_2$ . If  $\dot{M}_{ks}$  is the production rate of very small grains in annulus  $k$ , the space density of very small grains ejected from annulus  $k$  is

$$\rho_k = \frac{\dot{M}_{ks}}{4\pi a_k^2 v_{Kk} \sin \theta_k}, \quad (\text{A1})$$

where  $\theta_k$  is the opening angle defined by the vertical scale height of the very small grain population. The volume density  $\rho_k$  depends on the size distribution of very small grains,

$$\rho_k = \frac{4\pi\rho_g}{3} \int_{r_1}^{r_2} n_{0k} r^{3-\alpha}, \quad (\text{A2})$$

where  $\rho_g$  is the mass density of an individual grain. We adopt  $\alpha = 2.5$  and assume that grains have the same mass density as particles in the planetesimal grid. Solving the two equations for  $\rho_k$  yields the set of normalization constants  $n_{0k}$ . The normalization constants yield the volume density of the outflowing wind of very small grains as the sum of material ejected from all interior annuli

$$\rho_i = \sum_{k=1}^{k=i} \rho_k \quad (\text{A3})$$

To derive the optical depth of the very small grains, we adopt the geometric optics limit

$$\tau_{si} = 2\pi \int \int n(r, a) a^2 da dr, \quad (\text{A4})$$

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<sup>2</sup>Thébault, Augereau, & Beust (2003) perform a similar calculation in a single annulus and apply their results to the inner disk of  $\beta$  Pic.

where  $n = n_0 i r^{-\alpha}$  is the space number density. We solve the integral over the size distribution exactly and sum the optical depth through the planetesimal grid:

$$\tau_s = \frac{3(\sqrt{r_2/r_1} - 1)}{8\pi\rho_g r_2(1 - \sqrt{r_1/r_2})} \sum_{i=1}^N \left[ \sum_{k=1}^i \left( \frac{\dot{M}_k}{v_{Kk} h_k} \right) \left( \frac{1}{a_{b,k}} - \frac{1}{a_{b,k+1}} \right) \right], \quad (\text{A5})$$

where  $a_{b,k}$  is the inner boundary of an annulus centered at  $a_k$ .

Although the exact expression for the optical depth is a complicated function of grid variables, we derive a simple expression based on characteristic quantities averaged over the grid. For a reasonable grain population,  $r_1 = 0.01 \mu\text{m}$  and  $r_2 = 1 \mu\text{m}$ . The optical depth is

$$\tau_s \approx 0.056 \left( \frac{\dot{M}}{10^{21} \text{ g yr}^{-1}} \right) \left( \frac{30 \text{ AU}}{a} \right) \left( \frac{10 \text{ km s}^{-1}}{v_K} \right) \left( \frac{10^{-2}}{h} \right) \left( \frac{1.5 \text{ g cm}^{-3}}{\rho_g} \right) \left( \frac{10^{-4} \text{ cm}}{r_1} \right) \quad (\text{A6})$$

When the production rate of very small grains exceeds  $10^{22} \text{ g yr}^{-1}$ , the optical depth approaches unity. This rate corresponds to the ejection of 1–2 Earth masses every million years.

To derive the time evolution of the number density and optical depth for the larger grains, we consider three processes. Erosive collisions between particles in the planetesimal grid yield large grains with a size distribution  $N_C = N_0 r^{-2.5}$ . Erosive collisions between these particles remove material from the large grain population but maintain the same power law size distribution. Poynting-Robertson drag preferentially removes small grains and changes the size distribution. In each annulus, we begin with a set of discrete mass bins with minimum radius  $r_2$  and maximum radius  $r_{max}$ ;  $r_{max}$  is the radius of the smallest particle in the planetesimal grid. In most of our calculations,  $r_{max} = 1 \text{ m}$ . At  $t = 0$ , all of the large grain bins are empty. For each timestep, we add particles to the bins using  $N_C$  derived from the dust production rates  $\dot{M}_i$ . We compute the collision rates and outcomes for the largest mass bin in each annulus and scale this rate to derive collision rates and outcomes for smaller dust grains. We add or remove mass from each bin based on these rates. Finally, the change in particle number due to Poynting-Robertson drag is

$$\frac{dn_{ik}}{dt} = \lambda_{ijkl} n_{ik} \quad (\text{A7})$$

where  $\lambda_{ijkl}$  depends on the size, density, and radiative properties of the grains, the stellar flux, and the relative numbers of grains in adjacent annuli (e.g. Burns, Lamy, & Soter 1979). Because these properties change slowly with time, we assume  $\lambda_{ijkl}$  is constant during each timestep and integrate  $dn_{ik}/dt$  exactly to derive the amount of material dragged through the grid. This algorithm yields the number of grains in each mass bin in each annulus as a function of time. To derive the optical depth of the dust in each annulus, we assume the geometric optics limit and sum the optical depth of each mass bin.

Our simple dust collision algorithm yields the optical depths in planetesimals and planets, large dust grains, and very small dust grains in a discrete set of concentric annuli surrounding a star. The optical depth of grains dragged out of the innermost annulus by Poynting-Robertson

drag is small compared to the optical depth in all of the annuli; we ignore this contribution to the optical depth. Tests of the algorithm indicate that the optical depths are accurate to a factor of 1.5–2.0, which is adequate for our purposes.

To estimate the emergent luminosity from the disk, we assume that the central star is the only radiation source. We follow Kenyon & Hartmann (1987) and assume a spherical, limb-darkened star with radius  $R_0$ , luminosity  $L_0$ , and limb-darkening coefficient  $\epsilon_0 = 0.6$ . For a point  $P$  at the outer boundary of annulus  $i$  with height  $h_P$  above the disk midplane, rays from the star enter the annulus at a scale height  $h_{in}$  above (below) the midplane. We compute the length  $l$  of the path through the disk and derive the optical depth along this path as  $\tau_p = (l/\Delta a_i)\tau_i$ , where  $\Delta a_i$  is the width of the annulus. The radiation absorbed along this path is  $e^{-\tau_p}I_0$ , where  $I_0$  is the flux incident on the boundary of the annulus. Numerical integrations over the stellar surface and the vertical extent of an annulus yield the amount of flux absorbed by each annulus, which we convert to relative surface brightness. A final numerical integration over the radial extent of the disk yields the ratio of the disk luminosity to the stellar luminosity,  $L_D/L_0$ .

Although this algorithm is an improvement over our previous optically thin treatment of radiative transfer through a planetesimal disk, it has several limitations. The optical depth calculation does not distinguish between absorption and scattering. For a grain albedo  $\omega$ , the luminosity in scattered light is roughly  $\omega L_D/L_0$ ; the thermal luminosity is roughly  $(1-\omega)L_D/L_0$ . The algorithm also ignores multiple scattering and absorption. Because the vertical optical depth of the disk is  $\sim 10^{-3}$  or smaller in most cases, this approximation is satisfactory.

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